Unit IV

Single Phase Transformer

Construction

Principle

emf equation

Transformer efficiancy

UNIT - IV

Mohd. Murtaza F

Ideal when
(i) it has no core Losses (hysteresis & Eddy current loss)
(ii) Its winding resistence is zero
(iii) No leakage flux in primary & Secondary
(iv) Permeability of core is high so negligible current is required to set up a flux.

1

Transformer - I

1.1 PRINCIPLE OF TRANSFORMER OPERATION

A transformer is a static device which consists of two or more stationary electric circuits interlinked by a common magnetic circuit for the purpose of transferring electrical energy between them. The transfer of energy from one circuit to another takes place without a change in frequency.

Consider two coils 1 and 2 wound on a simple magnetic circuit as shown in Fig. 1.1. These two coils are insulated from each other and there is no electrical connection between them. Let T_1 and T_2 be the number of turns in coils 1 and 2 respectively. When a source of alternating voltage V_1 is applied to coil 1, an alternating current I_0 flows in it. This alternating current produces an alternating flux Φ_M in the magnetic circuit. The mean path of this flux is shown in Fig. 1.1 by

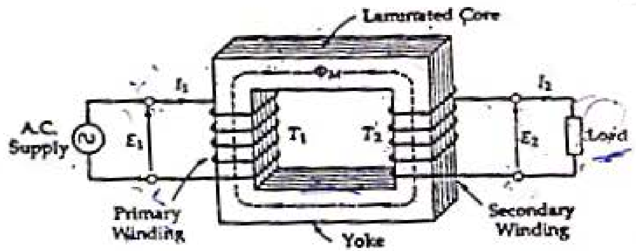


Fig. 1.1. *Arrangement of a simple transformer.*

the dotted line. This alternating flux links the turns T_1 of coil 1 and induces in them an alternating voltage E_1 by self-induction. Let us make the following simplifying *assumptions* for an *ideal transformer* :

(a) There are no losses either in the electric circuits or in the magnetic circuit.

voltage is lowered or raised. The turn ratio, or the induced voltage ratio, is called the transformation ratio and is denoted by the symbol a. Thus,

$$\boxed{a = \frac{E_1}{E_2} = \frac{T_1}{T_2}} \tag{1.3.5}$$

In a practical voltage transformer there is a very small difference between the terminal voltage and the induced voltage. Therefore, we can assume that $E_1 = V_1$, and $E_2 = V_2$. Equation (1.3.5) is modified as

$$\boxed{\frac{V_1}{V_2} = \frac{T_1}{T_2} = a} \tag{1.3.6}$$

If a voltage ratio or turns ratio is specified, this is always put in the order input : output, which is primary : secondary. It is to be noted from Eq. (1.3.6) that almost any desired voltage ratio can be obtained by adjusting the number of turns.

1.4 STEP-UP AND STEP-DOWN TRANSFORMERS

A transformer in which the output (secondary) voltage is greater than its input (primary) voltage is called a **step-up transformer.**

A transformer in which the output (secondary) voltage is less than its input (primary) voltage is called a **step-down transformer.**

The same transformer can be used as a step-up transformer or a step-down transformer depending on the way it is connected in the circuit. When the transformer is used as a step-up transformer, the low voltage winding is the primary. In a step-down transformer, the high-voltage winding is the primary.

A transformer may receive energy at one voltage and deliver it at the same voltage. Such a transformer is called a **one-to-one (1 : 1) transformer.** For a 1 : 1 transformer $T_1 = T_2$ and $|E_1| = |E_2|$. Such a transformer is used to isolate two circuits.

EXAMPLE 1.1. *A 3300/250 V, 50 Hz, single-phase transformer is built on a core having an effective cross-sectional area of 125 cm² and 70 turns on the low-voltage winding. Calculate (a) the value of the maximum flux density, (b) the number of turns on the high voltage winding.*

SOLUTION. $E_1 = 3300\text{ V},\ E_2 = 250\text{ V},\ f = 50\text{ Hz}$

$$A = 125\text{ cm}^2 = 125 \times 10^{-4}\text{ m}^2$$

$$E_2 = 4.44\,\Phi_m f T_2 = 4.44\,B_m A f T_2$$

$$B_m = \frac{E_2}{4.44\,A f T_2}$$

$$= \frac{250}{4.44 \times 125 \times 10^{-4} \times 50 \times 70} = 1.289\text{ teslas (T)}$$

$$\frac{E_1}{E_2} = \frac{T_1}{T_2}$$

$$T_1 = \frac{E_1}{E_2} \times T_2 = \frac{3300}{250} \times 70 = 924$$

EXAMPLE 1.2. *A transformer with 800 primary turns and 200 secondary turns is supplied from a 100 V a.c. supply. Calculate the secondary voltage and the volts per turn.*

SOLUTION. $T_1 = 800,\ T_2 = 200,\ V_1 = 100\text{ V}$

$$\frac{V_1}{V_2} = \frac{T_1}{T_2},\quad V_2 = V_1 \times \frac{T_2}{T_1}$$

$$= 100 \times \frac{200}{800} = 25\text{ V}$$

Volts per turn $= \dfrac{V_1}{T_1} = \dfrac{100}{800} = 0.125$

or volts per turn $= \dfrac{V_2}{T_2} = \dfrac{25}{200} = 0.125$

EXAMPLE 1.3. *A transformer with an output voltage of 4200 V is supplied at 230 V. If the secondary has 2000 turns, calculate the number of primary turns.*

SOLUTION. $V_2 = 4200\text{ V},\ V_1 = 230\text{ V},\ T_2 = 2000$

$$\frac{V_1}{V_2} = \frac{T_1}{T_2}$$

$$T_1 = \frac{T_2 V_1}{V_2}$$

$$= 2000 \times \frac{230}{4200} = 109.52\text{ turns}$$

In practice, it is not possible for a winding to have part of a turn (that is, turns cannot be fractional). Therefore, the number of turns should be a whole number. In our case we shall take $T_1 = 110$.

1.5 CONSTRUCTION OF SINGLE-PHASE TRANSFORMERS

A single-phase transformer consists of primary and secondary windings put on a magnetic core. Magnetic core is used to confine flux to a definite path. Transformer cores are made from thin sheets (called *laminations*) of high-grade silicon steel. The laminations reduce eddy-current loss and the silicon steel reduces hysteresis loss. The laminations are insulated from one another by heat resistant enamel insulation coating. L-type and E-type laminations are used. The laminations are built up into stack and the joints in the laminations are staggered to minimize airgaps (which require large exciting currents). The laminations are tightly clamped.

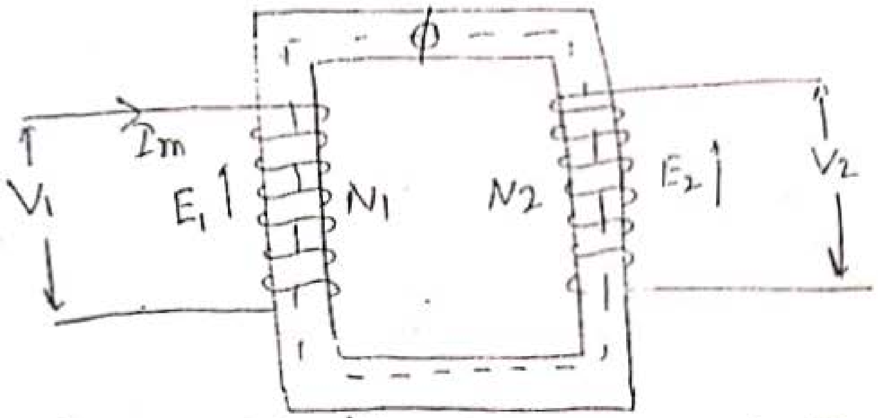
There are two basic types of transformer constructions, the core type and the shell type.

1.5.1 Core-type Construction

In the core-type transformer, the magnetic circuit consists of two vertical *legs or limbs* with two horizontal sections, called *yokes*. To keep the leakage flux to a minimum, half of each winding is placed on each leg of the core as shown in

Emf equation of a transformer

Consider that an alternating voltage V_1 of frequency f is applied to the primary of the transformer as shown in fig



The sinusoidal flux ϕ produced by the primary can be represented as

$$\phi = \phi_m \sin\omega t$$

The Instantaneous e.m.f e_1 induced in the primary is

$$e_1 = -N_1 \frac{d\phi}{dt} = -N_1 \frac{d}{dt}(\phi_m \sin\omega t)$$

$$= -N_1 \phi_m \cos\omega t \cdot \omega$$

$$= -\omega N_1 \phi_m \cos\omega t$$

$$= 2\pi f N_1 \phi_m \sin(\omega t - 90°)$$

it is clear from the above equation that maximum value of induced emf in the primary is

$$E_{m1} = 2\pi f N_1 \phi_m$$

[illegible] value E_1 of the primary emf is

$$E_1 = \frac{E_{m1}}{\sqrt{2}} = \frac{2\pi f N_1 \phi_m}{\sqrt{2}}$$

$$E_1 = 4.44 f N_1 \phi_m$$

Similarly $E_2 = 4.44 f N_2 \phi_m$

In an ideal Transformer $E_1 = V_1$ and $E_2 = V_2$

it is clear that $\frac{E_1}{E_2} = \frac{N_1}{N_2}$ or $\frac{E_1}{N_1} = \frac{E_2}{N_2}$

Voltage regulation of a Transformer :-

it is defined as the change in magnitude of the Secondary terminal voltage, expressed as a percentage of the secondary rated voltage, when load at a given power factor is reduced to zero, with primary applied voltage held constant.

V_2 = Secondary terminal voltage at any Load

E_2 = " " " " No "

Then at given power factor and specified load, the voltage regulation is given by

$$\text{Voltage regulation} = \frac{E_2 - V_2}{E_2} \times 100 \text{ in } \%$$

Transformer losses :→

There are mainly two types of losses in Transformer

(1) Iron or Core losses → Hysteresis losses (P_h)
(P_c) Eddy current losses (P_e)

$$P_c = P_h + P_e$$

(2) Copper loss or Ohmic or I^2R losses (P_i)

(1) Core losses (P_c) :→ The core loss P_c occuring in the transformer iron. Consists of two components. hysteresis loss P_h and eddy current loss P_e etc.

$$P_c = P_h + P_e$$

Here $P_h = k_h f B_m^x$

$$P_e = k_e f^2 B_m^2$$

where k_h = proportionality constant which depends upon the volume and quality of the core material and the units used.

k_e = proportionality constant whose value depends on the volume and resistivity of the core ...

B_m = maximum flux density in the core.

f = frequency of the alternating flux.

x = varies from 1.5 to 2.5, depending upon the magnetic properties of the core material (called Steinmetz's Constant)

(2) Copper or I^2R losses (P_l) :→ Copper losses is the I^2R loss which takes place in the primary and secondary winding because of the winding resistances.

Total Copper Loss in a Transformer = primary winding Copper loss + Secondary winding copper loss

$$P_l = I_1^2R_1 + I_2^2R_2$$

Since $I_1N_1 = I_2N_2$

$$I_1 = I_2\frac{N_2}{N_1}$$

$$P_l = I_2^2\left(\frac{N_2}{N_1}\right)^2R_1 + I_2^2R_2 = I_2^2\left(R_2 + \left(\frac{N_2}{N_1}\right)^2R_1\right) = I_2^2R_{e2}$$

$$\boxed{\text{or } P_l = I_1^2R_{e1} = I_2^2R_{e2}}$$

Numerical

Q:→ In a transformer, the core loss is found to be 52 watts at 40 Hz and 90 watts at 60 Hz: both losses being measured at the same peak flux density. Compute the hysteresis and eddy current losses at 50 Hz

Solution:→

$$P_c = K_h f B_m^x + K_e f^2 B_m^2$$

For constant flux density

$$P_c = k_h f + k_e f^2$$

where $k_h = K_h B_m^x$

$k_e = K_e B_m^2$

at 40 Hz $\quad 52 = k_h(40) + k_e(40)^2$

at 60 Hz $\quad 90 = k_h(60) + k_e(60)^2$

or

$$52 = 40k_h + 1600k_e$$

$$90 = 60k_h + 3600k_e$$

$$k_h = \frac{9}{10}, \quad k_e = \frac{1}{100}$$

Thus at 50 Hz

Hysteresis loss $P_h = k_h f = \frac{9}{10} \times 50 = 45$ watts

eddy current loss $P_e = k_e f^2$

Transformer efficiency:→ The efficiency of a Transformer (or any other device) is defined as the ratio of output power to input power. Thus

$$\text{Efficiency } \eta = \frac{\text{output power}}{\text{input power}}$$

$$\eta = \frac{V_2 I_2 \cos\theta_2}{V_2 I_2 \cos\theta_2 + P_c + I_2^2 r_{e2}} \quad \text{-----} \ (1)$$

where P_c = Total core loss

$I_2^2 r_{e2}$ = Total ohmic losses

$V_2 I_2$ = output VA

$\cos\theta_2$ = load Power factor

$$\eta = \frac{\text{Output Power}}{\text{Input power}} = \frac{\text{Input power} - \text{losses}}{\text{Input power}}$$

$$\eta = 1 - \frac{\text{losses}}{\text{Input power}}$$

Conditions for maximum efficiency:→ In eqn (1) P_c is constant and the load voltage V_2 remains practically constant. At a specified value of load p.f $\cos\theta_2$. The efficiency will be max when $\frac{d\eta}{dI_2} = 0$

$$\frac{d}{dt}\left(\frac{M}{N}\right) = \frac{(M)\frac{d}{dt}(N) - N\frac{d}{dt}(M)}{(N)^2}$$ ②

$$\frac{d\eta}{dI_2} = \frac{(V_2 I_2 \cos\theta_2 + P_c + I_2^2 r_{e2}) \times \frac{d}{dt}(V_2 I_2 \cos\theta_2) - (V_2 I_2 \cos\theta_2)\frac{d}{dt}(V_2 I_2 \cos\theta_2 + P_c + I_2^2 r_{e2})}{[V_2 I_2 \cos\theta_2 + P_c + I_2^2 r_{e2}]^2}$$

$$= 0$$

$$(V_2 I_2 \cos\theta_2 + P_c + I_2^2 r_{e2})(\cancel{V_2 \cos\theta_2}) = (\cancel{V_2} I_2 \cancel{\cos\theta_2})(V_2 \cos\theta_2 + 2 I_2 r_e$$

$$(V_2 I_2 \cos\theta_2 + P_c + I_2^2 r_{e2}) = I_2 (V_2 \cos\theta_2 + 2 I_2 r_{e2})$$

$$V_2 \cancel{I_2 \cos\theta_2} + P_c + I_2^2 r_{e2} = V_2 \cancel{I_2 \cos\theta_2} + 2 I_2^2 r_{e2}$$

$$\boxed{P_c = I_2^2 r_{e2}}$$

Constant core loss = variable ohmic loss

When energy efficiency is computed for a day of 24 hours, it is called all day efficiency

$$\text{All day } \eta = 1 - \frac{\text{Daily losses in kWh}}{\text{Daily input in kWh}}$$

All Day (or Energy) Efficiency :→

The primary of a distribution transformer is connected to the line for 24 hours a day. Thus the core losses occur for the whole 24 hours whereas copper losses occur only when the transformer is on load. The performance of a distribution transformer is represented by all-day or energy efficiency. Energy efficiency of a transformer is defined as the ratio of total energy output for a certain period to the total energy input for the same period. The energy efficiency can be calculated for any specified period. When the energy efficiency is calculated for a day of 24 hours it is called the all-day efficiency. All day efficiency is defined as the ratio of the energy output to the energy input taken over a 24-hour period.

$$\text{all day } \eta = \frac{\text{energy output for 24 hours}}{\text{energy input for 24 hours}}$$

$$= \frac{\text{output}}{\text{output} + \text{losses}}$$

$$\text{all day } \eta = 1 - \frac{\text{Daily Losses in kWh}}{\text{Daily input in kWh}}$$

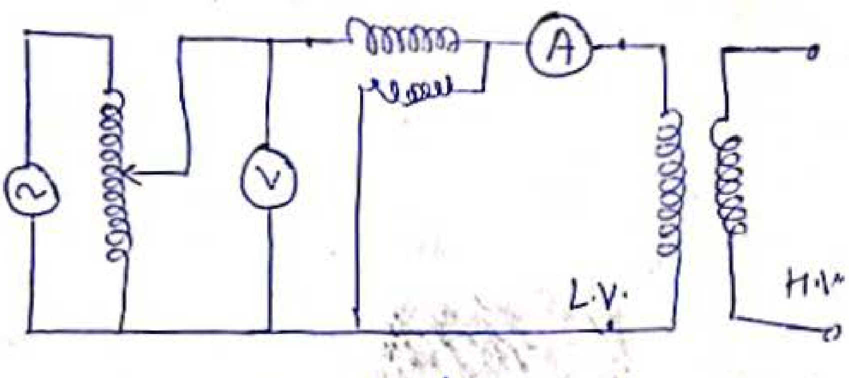
Testing of Transformer:→

There are two tests on a transformer, help to determine
(1) open circuit test
(2) Short " "

(i) The equivalent circuit parameter
(ii) The voltage regulation
(iii) efficiency.

(A) open circuit Test (No load Test):– it is used to determine shunt parameter of the equivalent circuit of the transformer and core loss of the Transformer.

In This test usualy High voltage (H.V.) side ~~kee~~ keep open and Test is conducted on the low voltage side of the transformer. in the L.V side the rated voltage is applied at the input terminal.



(a) circuit diagram for open-circuit Test for perform this test, the rated frequency voltage is applied to the primary i.e. low voltage side. is varied with the help of a variable ratio auto Transformer. When the voltage reading is equal to the rated voltage of the l.v winding, all the three reading are recorded.

The ammeter records the no load current or exciting current I_e.

The input power given by the wattmeter reading Consists of a core loss and ohmic loss.

Here ohmic loss (I^2R) losses are very small

So wattmeter reading can be taken as equal to Transformer core loss. (P_c)

V_1 = applied rated voltage on l.v side

I_e = exciting current (no load current)

P_c = Core loss

$$P_c = V_1 I_e \cos\theta_0$$

$$\therefore \cos\theta_0 = \frac{P_c}{V_1 I_e}$$

We know that

$$I_c = I_e \cos\theta_0 \qquad I_m = I_e \sin\theta_0$$

and $$I_c = \frac{P_c}{V_1}$$

$\therefore$ Core loss resistance

$$R_{CL} = \frac{V_1}{I_c} = \frac{V_1}{I_e \cos\theta_0} = \frac{V_1^2}{V_1 I_e \cos\theta_0} = \frac{V_1^2}{P_c}$$

also $P_c = I_c^2 R_{CL}$

$$R_{CL} = \frac{P_c}{I_c^2} = \frac{P_c}{(I_e \cos\theta_0)^2}$$

Magnetization reactance

$$X_{mL} = \frac{V_1}{I_m} = \frac{V_1}{I_e \sin\theta_0}$$

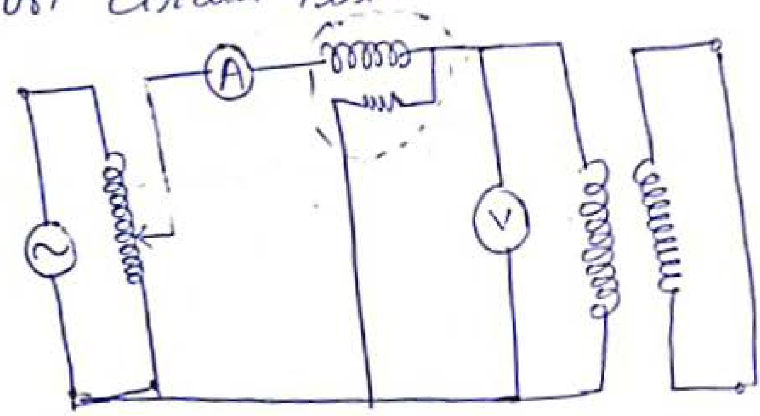
Thus the open circuit Test gives the following information

(i) Core loss at rated voltage and frequency

(ii) The shunt branch parameter of the equivalent circuit R_c and X_m

(iii) Turns ratio of Transformer

② Short Circuit Test



This test is conduct to determine the series parameter of the equivalent circuit ~~that is required to determine the~~ equivalent circuit parameter and copper losses of the Transformer.

In this test usualy low voltage winding kept short circuit and test is conduct on the H.V. side in which the variable voltage supply is connected at the input terminals. The rated current is applied at the input by adjusting the voltage and take the reading of all meter at that time. The voltmeter, ameter and wattmeter give V_{sc}, I_{sc} and P_{sc} respectively

$$Z_{sc} = \frac{V_{sc}}{I_{sc}}$$

equivalent resistance refered to h.v side

$$r_{e11} = \frac{P_{sc}}{I_{sc}^2}$$

and equivalent leakage reactance referred to h.v side

$$x_{e11} = \sqrt{Z_{e11}^2 - r_{e11}^2}$$

Hence for equivalent circuit parameter

$$r_1 = r_2 = \frac{1}{2} r_e$$

$$x_1 = x_2 = \frac{1}{2} x_e$$

Thus the short circuit Test gives the following information

(i) Ohmic loss at rated current and frequency

(ii) The equivalent resistance and equivalent leakage reactance.

20kVA, 2500/250V, 50Hz, Single Phase transformer gave the following test result

Open circuit Test (on L.V side) : 250V, 1.4A, 105 watts

Short Circuit Test (on HV side): 104V, 8A, 320 watts

Compute the parameters of the ~~following~~ approximate equivalent circuit referred to high voltage and low voltage side

Solution: → given data is

for O.C. Test → $V_1 = 250V$
$I_e = 1.4A$
$P_c = 105\,W$

for S.C. Test – $V_{sc} = 104V$
$I_{sc} = 8A$
$P_{sc} = 320$ watts.

Solve for O.C. Test

$$P_c = V_1 I_e \cos\theta_0$$

$$\cos\theta_0 = \frac{P_c}{V_1 I_e} = \frac{105}{250\times 1.4} = 0.3$$

$$\cos\theta_0 = 0.3$$

$$\theta_0 = 72.55^0$$

So $\sin\theta_0 = 0.954$

Now

$$I_c = I_e \cos\theta_0 = \cancel{1.4 \times \cos 0.3} = 1.4\times 0.3 = 0.42A$$

$$I_m = I_e \sin\theta_0 = \cancel{1.4 \sin 0.954} = 1.4 \times 0.954 = 1.336A$$

Hence $R_{cL} = \frac{V_1}{I_c} = \frac{250}{0.42} = 595\Omega$

$$X_{mL} = \frac{V_1}{I_m} = \frac{250}{1.336} = 187\Omega$$